FERMATEAN FUZZY SOFT ROUGH SETS AND ITS APPLICATION IN DECISION MAKING

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ABSTRACT

Rough set theory is an extension of set theory to study the data with incomplete information. Fuzzy set theory is an another tool to deal uncertainity which is different from rough set theory. Soft set theory is a relatively new approach to discuss vagueness. These three theories are mathematical tools for dealing with uncertainties and are closely related and can be combined in a truthful way. In this paper a discussion of the combinations of fermatean fuzzy set, rough set and soft set are done. It aims to introduce two new notions that are soft rough fermatean fuzzy set and fermatean fuzzy soft rough sets. Examples for both soft rough fermatean fuzzy set and fermatean fuzzy soft rough sets is provided to illustrate the developed approach.

Keywords: Soft set, rough set, Fermatean fuzzy set, Fermatean fuzzy soft rough set, Pythagorean fuzzy set, Pythagorean fuzzy soft rough set,

INTRODUCTION

Many well-known theories have been introduced for exploration of uncertainties in last few years to deal with the complicated models in engineering, economics, medical sciences, social sciences. Out of these theories, fuzzy set theory (L A Zadeh, 1965), soft set theory (Molodtsov, 1999), rough set theory (Pawlak, 1982) are the most popular theories.

Theory of Rough sets was an extension of the set theory to study the data with incomplete information. In rough set theory a subset of the universe is described by two approximations, called lower and upper approximations. The lower approximation of a set is the union of all equivalence classes contained in the set, whereas the upper approximation is the union of those classes which have non empty intersection with the set. Equivalence classes are the basic building blocks in rough set theory, for upper and lower approximations. A partition of a set induces an equivalence relation and vice versa. Therefore, we can view the properties of a rough set either by the partition of the set or by the equivalence relation.

The soft set theory offers a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. This theory is a relatively new approach to discuss vagueness.

In soft set theory membership is decided by adequate parameters, rough set theory deals with equivalence classes, whereas fuzzy set theory depends upon grade of membership. Although three theories are quite distinct yet deal with vagueness. Joint application of these theories may result in a fruitful way. There exist many complications in soft set theory mentioned in (Molodtsov, 1999). To overcome these complications the new approach of SS was originated by Molodtsove, which is different from the other existing theories due to its parameterization tools. Soft set theory handles the vague and uncertain knowledge easily and it is free from the inherent complications. Due to these characteristics, the theory of Soft Set is famous among scholars in the recent era. In the recent scenario, Soft Set theory is one of the most significant areas of research. Maji (Maji et al., 2003) introduced various operations on Soft Sets. Through these operations, the theoretical study of Soft Set becomes improved and emerges very rapidly in research area. on the basis of (Maji et al., 2003) , (Ali et al., 2009) improved the existing literature and introduced some new operations on soft sets. They developed the concept of complement and asserted that certain De Morgans's laws are satisfied on the basis of these newly developed operations on Soft Sets. The generalization of Soft Set is improving very rapidly in the research area to hybridize the different mathematical structures with Soft Set. The combined structure of fuzzy set, soft set and rough set were presented by (Irfan Ali, 2011)

A fuzzy set is a class of objects with continuum of grade of membership. Such a set is characterized by membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

Intuitionistic fuzzy set (Atanassov K T, 1986) consist of two functions which are the generalization of fuzzy set and like a fuzzy set, these two functions also map on unit closed interval. Here, the first function represents the membership grade and the second function represents non-membership grade. Intuitionistic fuzzy set set has a limitation that is the sum of both these functions does not exceed 1. Intuitionistic fuzzy set set plays a vital role for tackling vagueness and uncertainty. Many authors developed the new notions by combining rough set and Soft sets with Intuitionistic fuzzy set set and proposed the new results.

Yager (Yager, 2013) introduced Pythagorean fuzzy set, the square sum of membership and non- membership grades must not exceed 1, which got more attention of scholars in the recent era. (Hussain et al., 2019)proposed the concept of rough Pythagorean fuzzy ideals in semigroups. (Hussain et al., 2020) introduced the concepts of soft rough Pythagorean fuzzy set (SRPFS) and Pythagorean fuzzy soft rough set (PFSRS)

(Senapati & Yager, 2020) proposed Fermatean fuzzy sets (FFS), the cube sum of membership and non- membership grades must not exceed 1. He compared Fermatean fuzzy sets with Pythagorean fuzzy sets and intuitionistic fuzzy sets. (Muhammed, 2022) proposed the concept of rough Fermatean fuzzy ideals in semigroups. By viewing existing literature, it is clear that there does not exist any concepts of soft rough Fermatian fuzzy set (SRFFS) and Fermatian fuzzy soft rough set (FFSRS).

PRELIMINARIES

This section consists of a brief review of Intuitionistic fuzzy set(IF set), Fermatean fuzzy set (FFS), rough soft set (RSS), soft set(SS) and soft rough set.

Definition 2.1 (Atanassov K T, 1986)

For a universal set U, an IF set on U is denoted and defined as

$$\mathsf{f}_F = \{(k, \mu_{I_F}, \psi_{I_F}) \mid k \in U\}$$

Where $\mu_{\mathsf{f}_F}: U \rightarrow [0,1]$ represents the membership degree and $\psi_{\mathsf{f}_F}: U \rightarrow [0,1]$ represents non membership degree of $k \in U$ to the set f_F satisfying $0 \leq \mu_{I_F} + \psi_{I_F} \leq 1$.

Also $\pi_{\mathsf{f}_F}(k) = 1 - \mu_{\mathsf{f}_F} + \psi_{\mathsf{f}_F}$ is known as indeterminacy or hesitancy.

Definition 2.2 (Yager, 2013)

Let U be the universal set. A fermatean fuzzy set on U is denoted and defined as

$$\mathcal{F}_{\mathcal{F}} = \{(k, \mu_{\mathcal{F}_{\mathcal{F}}}, \psi_{\mathcal{F}_{\mathcal{F}}}) \mid k \in U\}$$

Where $\mu_{\mathcal{F}_{\mathcal{F}}}: U \rightarrow [0,1]$ represents the membership degree and $\psi_{\mathcal{F}_{\mathcal{F}}}: U \rightarrow [0,1]$ represents non membership degree of $k \in U$ to the set $\mathcal{F}_{\mathcal{F}}$ satisfying $0 \leq \mu^3_{\mathcal{F}_{\mathcal{F}}} + \psi^3_{\mathcal{F}_{\mathcal{F}}} \leq 1$.

Also $\pi_{\mathcal{F}_{\mathcal{F}}}(k) = \sqrt{1 - (\mu^3_{\mathcal{F}_{\mathcal{F}}} + \psi^3_{\mathcal{F}_{\mathcal{F}}})}$ is known as indeterminacy or hesitancy.

Basic operations union, intersection, complement on fermatean fuzzy sets are defined by (Senapati & Yager, 2020) and are given as

Consider two fermatean fuzzy sets

$$\mathcal{F}_{\mathcal{F}1} = \{(k, \mu_{\mathcal{F}_{\mathcal{F}1}}, \psi_{\mathcal{F}_{\mathcal{F}1}}) \mid k \in U\}$$

and

$$\mathcal{F}_{\mathcal{F}_2} = \{(k, \mu_{\mathcal{F}_{\mathcal{F}2}}, \psi_{\mathcal{F}_{\mathcal{F}2}}) \mid k \in U\}$$

$$\mathcal{F}_{\mathcal{F}1} \cup \mathcal{F}_{\mathcal{F}2} = \{(k, max\,(\mu_{\mathcal{F}_{\mathcal{F}1}}, \mu_{\mathcal{F}_{\mathcal{F}2}}), min(\psi_{\mathcal{F}_{\mathcal{F}1}}, \psi_{\mathcal{F}_{\mathcal{F}2}}) \mid k \in U\}$$

$$\mathcal{F}_{\mathcal{F}1} \cap \mathcal{F}_{\mathcal{F}2} = \{(k, min\,(\mu_{\mathcal{F}_{\mathcal{F}1}}, \mu_{\mathcal{F}_{\mathcal{F}2}}), max(\psi_{\mathcal{F}_{\mathcal{F}1}}, \psi_{\mathcal{F}_{\mathcal{F}2}}) \mid k \in U\}$$

$$\mathcal{F}^c_{\mathcal{F}} = \{(k, \psi_{\mathcal{F}_{\mathcal{F}}}, \mu_{\mathcal{F}_{\mathcal{F}}})|k \in U\}$$

Definition 2.3 (Molodtsov, 1999)

Let U be a universal set and E be the initial set of parameters. Suppose that a function $F: E \to P(U)$, then the pair (F, E) is known to be a soft set on U, where $P(U)$ represents family of all subsets of U.

Definition 2.4 (Cagman N, 2010)

Consider a soft set (F, E) on U, then a relation Ω from $U \times E$ is known to be a crisp soft relation from a set U to E and is given as $\Omega = \{< (k, x), \mu_\Omega(k, x) > |(k, x) \in U \times E\}$

Where $\mu_\Omega : U \times E \to [0,1]$ and $\mu_\Omega(k, x) = \begin{cases} 1, & (k, x) \in \Omega \\ 0, & (k, x) \notin \Omega \end{cases}$

Definition 2.5 (Maji PK, Biswas R, 2001)

Let U be a universal set and E be the initial set of parameters. Suppose that a function $F: E \to P(U)$, then the pair (F, E) is known to be a fuzzy soft set on U, where $P(U)$ represents family of all fuzzy subsets of U.

Definition 2.6 (Cagman N, 2010)

Consider a fuzzy soft set (F, E) on U, then a relation Ω from $U \times E$ is known to be a fuzzy soft relation from a set U to E and is given as

$$\Omega = \{< (k, x), \mu_\Omega(k, x) > |(k, x) \in U \times E\}$$

Where $\mu_\Omega: U \times E \to [0,1]$ and $\mu_\Omega(k, x) = \mu_{F(x)}(k)$

Definition 2.7 (YY., 1998; YY, 1998)**:**

Consider a universal set U and $\Omega \subsetneq U \times U$ be a crisp relation. Consider the mapping $\Omega^*: U \to P(U)$ defined by $\Omega^*(k) = \{x \in U|(k, x) \in \Omega\}$. Then (u, Ω) is said to be an approximation space. Consider a nonempty subset $F \subsetneq U$, then the lower and upper approximations of F with respect to (u, Ω) are denoted by $\underline{\Omega}$(F) and $\overline{\Omega}(F)$ are defined as follows

$$\underline{\Omega}(F) = \{k \in U|\Omega^*(k) \subsetneq F\}$$

$$\overline{\Omega}(F) = \{k \in U|\Omega^*(k) \cap U \neq \phi\}$$

Then the pair $(\underline{\Omega}(F), \overline{\Omega}(F))$ is the crisp soft rough set where $\underline{\Omega}(F) \neq \overline{\Omega}(F)$.

1. Soft rough Fermatean fuzzy set (SRFFS)

In this section we will present the concept of SRFFS by combining the crip soft relation from U to E with the soft rough fermatean fuzzy approximation.

First we are discussing the soft rough fermatean fuzzy approximation operators.

Definition 3.1

Consider a crisp soft approximation space (U, E, Ω).

Let $\mathcal{F_F} = \{(k, \mu_{\mathcal{F_F}}(k), \psi_{\mathcal{F_F}}(k)) \mid k \in E\}$, where $\mathcal{F_F}$ is the fermatean fuzzy set.

Then the lower and upper soft approximations of F with respect to (U, E, Ω) are represented by $\underline{\Omega}(F_F)$ and $\overline{\Omega}(F_F)$ and are defined as

$$\underline{\Omega}(F_F) = \{ (k, \mu_{\underline{\Omega}(F_F)}(k), \psi_{\underline{\Omega}(F_F)}(k)) \mid k \in U\}$$

$$\overline{\Omega}(F_F) = \{ (k, \mu_{\overline{\Omega}(F_F)}(k), \psi_{\overline{\Omega}(F_F)}(k)) \mid k \in U\}$$

Where $\mu_{\underline{\Omega}(F_F)}(k) = \wedge_{k_1 \in \Omega^*(k)} (\mu_{\mathcal{F_F}}(k_1))$ and

$\psi_{\underline{\Omega}(F_F)}(k) = \vee_{k_1 \in \Omega^*(k)} (\psi_{\mathcal{F_F}}(k))$

Which satisfies the condition $0 \leq (\mu_{\underline{\Omega}(F_F)}(k))^3 + (\psi_{\underline{\Omega}(F_F)}(k))^3 \leq 1$ and

$\mu_{\overline{\Omega}(F_F)}(k) = \vee_{k_1 \in \Omega^*(k)} (\mu_{\mathcal{F_F}}(k_1))$ and

$\psi_{\overline{\Omega}(F_F)}(k) = \wedge_{k_1 \in \Omega^*(k)}(\psi_{\mathcal{F_F}}(k))$

Which satisfies the condition $0 \leq (\mu_{\overline{\Omega}(F_F)}(k))^3 + \psi_{\overline{\Omega}(F_F)}(k))^3 \leq 1$

The pair $(\underline{\Omega}(F_F), \overline{\Omega}(F_F))$ is known as Soft rough fermatean fuzzy set with respect to (U, E, Ω) where $\underline{\Omega}(F_F) \neq \overline{\Omega}(F_F)$. The operators $\underline{\Omega}(F_F), \overline{\Omega}(F_F): FFS(E) \to FFS(U)$ are known as lower and upper soft rough fermatean fuzzy approximation operators with respect to (U, E, Ω).

Example 3.1

Suppose $U = \{k_1, k_2, k_3, k_4, k_5\}$ and $E = \{x_1, x_2, x_3, x_4\}$ be the initial set of parameters. A soft set (F, E) over U is defined as

$$F(x_1) = \{k_1, k_4\}, F(x_2) = \emptyset, F(x_3) = \{K_2, k_3\}, F(x_4) = \mathrm{U}$$

Define a crisp soft relation Ω on $U \times E$,

$\Omega = \{(k_1, x_1), (k_2, x_4), (k_2, x_3), (k_3, x_2), (k_4, x_1), (k_4, x_3), (k_5, x_1), (k_1, x_3), (k_3, x_4)\}$

$$\Omega^*(k_1) = \{x_1, x_3\}$$
$$\Omega^*(k_2) = \{x_3, x_4\}$$
$$\Omega^*(k_3) = \{x_2, x_4\}$$
$$\Omega^*(k_4) = \{x_1, x_3\}$$
$$\Omega^*(k_5) = \{x_1\}$$

Define a fermatean fuzzy set on E as,

$$F_F = (x_1, 0.9, 0.5), (x_2, 0.9, 0.6), (x_3, 0.8, 0.7), (x_4, 0.8, 0.6)$$

Lower approximation space is

$$\underline{\Omega}(F_F) = \{(k_1, 0.8, 0.7), (k_2, 0.8, 0.7), (k_3, 0.8, 0.6), (k_4, 0.8, 0.7), (k_5, 0.9, 0.5)\}$$

Upper approximation space

$$\overline{\Omega}(F_F) = \{(k_1, 0.9, 0.5), (k_2, 0.8, 0.6), (k_3, 0.9, 0.6), (k_4, 0.9, 0.5), (k_5, 0.9, 0.5)\}$$

2. Fermatean fuzzy soft rough set (FFSRS)

In this section, we originate a new notion of FFSRS on the basis of Pythagorean fuzzy soft rough set (Hussain et al., 2020).

Definition 4.1

Let E be the set of parameters and U be the universal set. Then (F, E) is said to be Fermatean fuzzy soft set over U, if $F: E \rightarrow FFS(U)$ such that $\forall x \in E$,

$$F(x) = \{(k, \ \mu_{F(x)}(k), \psi_{F(x)}(k)) \mid k \in U\} \in FFS(U)$$

where $\mu_{F(x)}(k), \psi_{F(x)}(k) \in [0,1]$ represents membership and non-membership grade which satisfies $\mu_{F(x)}(k))^3 + \psi_{F(x)}(k))^3 \leq 1$

Definition 4.2

let E be the set of parameters and U be the universal set. Suppose Ω be the fuzzy soft relation on U. Then the triplet (U, E, Ω) is said to be fuzzy soft approximation space.

For any $F_F = \{(x, \mu_{\mathcal{F}_{\mathcal{F}}}(x), \psi_{\mathcal{F}_{\mathcal{F}}}(x)) \mid x \in E\} \in FFS(E)\}$ then the lower and upper approximations of F_F with respect to (U, E, Ω) are represented by $\underline{\Omega}(F_F)$ and $\overline{\Omega}(F_F)$ and are defined as

$$\underline{\Omega}(F_F) = \{ (k, \mu_{\underline{\Omega}(F_F)}(k), \psi_{\underline{\Omega}(F_F)}(k)) \mid k \in U\}$$

$$\overline{\Omega}(F_F) = \{ (k, \mu_{\overline{\Omega}(F_F)}(k), \psi_{\overline{\Omega}(F_F)}(k)) \mid k \in U\}$$

where $\mu_{\underline{\Omega}(F_F)}(k) = \wedge_{x\in E} \{ (1- \mu_{\Omega}(k,x)) \vee \mu_{\mathcal{F}_{\mathcal{F}}}(x)\}$ and

$$\psi_{\underline{\Omega}(F_F)}(k) = \bigvee_{x\in E} \{ \mu_{\Omega}(k,x) \wedge \psi_{\mathcal{F}_{\mathcal{F}}}(x)\}$$

also

$\mu_{\overline{\Omega}(F_F)}(k) = \vee_{x\in E} \{ (\mu_{\Omega}(k,x)) \wedge \psi_{\mathcal{F}_{\mathcal{F}}}(x)\mu_{\mathcal{F}_{\mathcal{F}}}(x)\}$ and

$$\psi_{\overline{\Omega}(F_F)}(k) = \bigwedge_{x\in E} \{ (1- \mu_{\Omega}(k,x)) \vee \psi_{\mathcal{F}_{\mathcal{F}}}(x)$$

Then the pair $(\underline{\Omega}(F_F), \overline{\Omega}(F_F))$ is known as fermatean fuzzy soft rough set (FFSRS) of F_F with respect to (u, E, Ω). where $\underline{\Omega}(F_F) \neq \overline{\Omega}(F_F)$. The operators $\underline{\Omega}(F_F), \overline{\Omega}(F_F): FFS(E) \rightarrow FFS(U)$ are known as lower and upper fermatean fuzzy soft rough approximation operators with respect to (U, E, Ω).

Here we will show that $\underline{\Omega}(F_F), \overline{\Omega}(F_F) \in FFS(U)$

$$\mu_{\underline{\Omega}(F_F)}(k)^3 + \psi_{\underline{\Omega}(F_F)}(k)^3$$

$$= \bigwedge_{x\in E}\{1- \mu_{\Omega}(k,x)^3) \vee \mu_{\mathcal{F}_{\mathcal{F}}}(x)^3\} + \bigvee_{x\in E} \{ \mu_{\Omega}(k,x)^3 \wedge \psi_{\mathcal{F}_{\mathcal{F}}}(x)^3\}$$

$$= 1 - \bigvee_{x\in E} \{ \mu_{\Omega}(k,x)^3 \vee 1 - \mu_{\mathcal{F}_{\mathcal{F}}}(x)^3\} + \bigvee_{x\in E} \{ \mu_{\Omega}(k,x)^3 \wedge \psi_{\mathcal{F}_{\mathcal{F}}}(x)^3\}$$

$$\leq 1 - \bigvee_{x\in E} \{ \mu_{\Omega}(k,x)^3 \vee 1 - \mu_{\mathcal{F}_{\mathcal{F}}}(x)^3\} + \bigvee_{x\in E} \{ \mu_{\Omega}(k,x)^3 \wedge 1 - \mu_{\mathcal{F}_{\mathcal{F}}}(x)^3\} = 1$$

Therefore $\underline{\Omega}(F_F) \in FFS(U)$. Similarly, we can show that $\overline{\Omega}(F_F) \in FFS(U)$

The operators $\underline{\Omega}(F_F), \overline{\Omega}(F_F): FFS(E) \rightarrow FFS(U)$ are called lower and upper approximation operators with respect to (U, E, Ω)

Example 4.1

Let $U = \{k_1, k_2, k_3, k_4, k_5\}$ and $E = \{x_1, x_2, x_3, x_4\}$ be the initial set of parameters. Consider a fuzzy soft set (F, E) over U, a fuzzy soft relation Ω from $U \times E$ is defined as

Ω	x_1	x_2	x_3	x_4
k_1	0.8	0.6	0.5	0.7
k_2	0.9	0.3	0.4	0.1
k_3	0.3	0.7	0.8	0.6
k_4	0.2	0.8	0.3	0.2
k_5	0.5	0.9	0.6	0.7

Define a fermetean fuzzy set

$$F_F = (x_1, 0.9, 0.5), (x_2, 0.6, 0.9), (x_3, 0.8, 0.7), (x_4, 0.8, 0.6)$$

Then $\underline{\Omega}(F_F), \overline{\Omega}(F_F)$ are

$$\underline{\Omega}(F_F) = (k_1, 0.6, 0.6), (k_2, 0.7, 0.5), (k_3, 0.6, 0.7), (k_4, 0.6, 0.8), (k_5, 0.6, 0.8)$$

$$\overline{\Omega}(F_F) = (k_1, 0.6, 0.6), (k_2, 0.5, 0.7), (k_3, 0.7, 0.6), (k_4, 0.6, 0.7), (k_5, 0.6, 0.6)\}$$

APPLICATION OF FERMATEAN FUZZY SOFT ROUGH SETS IN DECISION MAKING

The technique for the Decision making process is constructed on the approach of FFSRS . It is more effective to deal with DM problem with the evaluation of Fermatean fuzzy information based on SRFFS and FFSRS models than DM problems with the evaluation of SRPFS and PFSRS models.

Let (U, E, R) be a fuzzy soft approximation space, where U is the universe of the discourse, E is the parameter set, and R is a fuzzy soft relation on U×E. Then we can give an algorithm based on FF soft rough sets with five steps

- Step 1: construct a fuzzy soft relation R from U to E, or fuzzy soft set (F, E) over U
- Step 2: Construct an optimum normal decision object A on the basis of assumption, according to different needs of the decision maker.
- Step 3: we can compute the FF soft rough approximation operators $\underline{\Omega}(A) and\, \Omega(A)$ of the optimum normal decision object A
- Step 4: By the ring sum operation, we can compute the choice set. we

should take the object $u_j \in$U in universe U with the maximum choice value as the optimum decision for the given decision making problem.

- Step 5: Go back to the second step and change decision object so that the final decision is only one, when there exist too many "optimal choices" to be chosen

Example

Suppose that U= $\{u_1, u_2, u_3, u_4, u_5\}$ is the set of five houses under consideration of a decision maker to purchase. Let E be a parameter set, where E= $\{e_1, e_2, e_3, e_4, e_5\}$= {expensive; beautiful; size; location}. Mr. X wants to buy the house which qualifies with the parameters of E to the utmost extent from available houses in U. Assume that Mr. X describes the "attractiveness of the houses" by constructing a fuzzy soft set (F, E) which is a fuzzy soft relation from U to E. And it is presented by a table as in the following form:

R	e_1	e_2	e_3	e_4
u_1	0.7	0.1	0.5	0.3
u_2	0.2	0.5	0.8	0.7
u_3	0.3	0.3	0.1	0.6
u_4	0.5	0.5	0.2	0.4
u_5	0.5	0.4	0.1	0.8.

Now suppose that Mr. X gives the optimum normal decision object A which is a Fermatean Fuzzy subset defined as follows:

A= $(e_1, 0.9, 0.5), (e_2, 0.6, 0.9), (e_3, 0.8, 0.7), (e_4, 0.8, 0.6)$

$\underline{\Omega}(A) = \{(u_1, 0.6, 0.6), (u_2, 0.7, 0.5), (u_3, 0.6, 0.7), (u_4, 0.6, 0.8), (u_5, 0.6, 0.8)\}$

$\overline{\Omega}(A) = \{(u_1, 0.6, 0.6), (u_2, 0.5, 0.7), (u_3, 0.7, 0.6), (u_4, 0.6, 0.7), (u_5, 0.6, 0.6)\}$

$\overline{\Omega}(A) \oplus \underline{\Omega}(A)$=

$\{(u_1, 0.84, 0.36), (u_2, 0.85, 0.35), (u_3, 0.98, 0.42), (u_4, 0.84, 0.56), (u_5, 0.84, 0.48)\}$

Obviously optimum solution is u_3. Mr. X will buy the house u_3.

1. CONCLUSION

The theories of rough set, soft set, Intuitionistic fuzzy set, Pythagorean fuzzy set and Fermatean Fuzzy Set all are important mathematical tools for dealing with uncertainties. In this paper, we have presented two new concepts: SoftRough Fermatean Fuzzy Set and Fermatean Fuzzy Soft Rough Set. By combining crisp soft relation with soft rough approximation space, soft rough fermatean fuzzy approximation operators are defined. Also a new notion of Fermatean fuzzy soft rough set on the basis of Pythagorean fuzzy soft rough set is introduced. For that first we define a fermatean fuzzy soft set. Using a fuzzy soft relation, a fuzzy soft approximation space is defined and from that fermatean fuzzy soft rough approximation operators were defined.

These concepts can be very helpful in decision making problems. This method provides more freedom for the decision makers to the selection of membership and non-membership degrees as compared to existing literature and also it is more superior than the existing one

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